15/ENG06/017

If $y = e^{2x+x}$

$U = x^2 + x$

$\frac{\Delta u}{\Delta x} = 2x+1$

$y = e^{u}$

$\frac{\Delta y}{\Delta x} = e^{u}$

$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u} \times \frac{\Delta u}{\Delta x}$

$= e^{u} \times 2x+1$

$2x+1e^{u} \quad U = x^2+x$

$\frac{\Delta y}{\Delta x} = 2x+1e^{x^2+x}$

$\frac{\Delta^2 y}{\Delta x^2} = 2e^{x^2+x} + (2x+1)(2x+1)e^{x^2+x}$

$\frac{\Delta^2 y}{\Delta x^2} = 2e^{x^2+x} + 4x^2 + 4x + 1 + e^{x^2+x}$

$y'' = \frac{\Delta^2 y}{\Delta x^2} \quad y' = \frac{\Delta y}{\Delta x} \quad y = e^{x^2+2}$

$y'' = y'(2x+1) + 2y$

$y'' = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$y' = (2x+1) = (2x+1)(2x+1)e^{x^2+x}$

$= 4x^2 + 4x + 1e^{x^2+x}$

$2y = 2e^{x^2+x}$

$y'(2x+1) + 2y = 2e^{x^2+x} + 4x^2 + 4x + 1e^{x^2+x}$

$= 2e^{2x+x} + 4x^2 + 4x + 1e^{x^2+x}$

$$\underbrace{y''}_{w_1} = \underbrace{y'(2x+1)}_{w_2} + \underbrace{2y}_{w_2}$$

w_1 +1

$U = y''$ $\quad V = 1$

$U^n = y^{n+2}$ $\quad U = 0$

$= y^{n+2} - 1 + 0$

w_2

$U = y'$ $\quad V = 2x+1$

$U^n = y^{n+1}$ $\quad V = 0$

$(2[y^n - 1] + 0)$

$w_1 = w_2 + w_3$

$y^{n+1} = y^{n+1}(2x+1) + 2n(y^n) + 2y^n$

$= y^{n+1}(2x+1) + 2(n+1)y^n$

2a) ~~Using the~~

$y = x^3 e^{4x}$

determine $y^{(5)}$

$y^{(5)} = U^{(5)}V + 5U^4V' + 10U^3V'' + 10w^2V^3 + 5UV^4$

$= 4^5 \cdot e^{4x} \cdot x^3 + 5(4^4 e^{4x} \cdot 3x^2) + 10(4^3 e^{4x} 6x)$

$+ 5(4^2 e^{4x} \cdot 6) + 0$

$= 1024 e^{4x} x^3 + 1280 e^{4x} 3x^2 + 640 e^{4x} \cdot 6x$

$+ 80 e^{4x} \cdot 6$

$= 1024 e^{4x} x^3 + \cancel{3840 e^{4x} x^2}\ 3840 e^{4x} x^2 +$

$3840 e^{4x} x + 480 e^{4x}$

$$x^2 \frac{\Delta y^2}{\Delta x^2} + \frac{x\,dy}{\Delta x} + y = 0$$

$x^2 y'' + xy' + y = 0$

$\downarrow\ w_1 \qquad \downarrow\ w_2 \qquad \downarrow\ w_3$

w_1

$U = y''$ $\quad V = 8x^2$

$U^n = y^{n+2}$ $\quad V' = 2x$

$U^{n-1} = y^{n+1}$ $\quad V'' = 2$

$= y(n+2)(x^2) + n(y^{n+1})2x + n(n-1)\ y^n \cdot 2 = 0$

$= x^2 y^{(n+2)} + 2nx(y^{n+1}) + n(n-1)y^n$

w_2

$w = y'$ $\quad V = x$

$U^n = y^{n+1}$ $\quad V = 1$

$U^{n-1} = y^n$

$= y^{n+1} \cdot x + n y^n + 0$

w_3

$U = y$ $\quad V = 1$

$U^n = y^n$ $\quad V' = 0$

$= y^n$

$w_1 + w_2 + w_3 = 0$

$x^2 y^{n+2} + 2nxy^{n+1} + (n^2 - n)y^n + xy^{n+1} + ny^n$

$+ y^n$

$= x^2 y^{n+2} + 2n + 1(xy^{n+1}) + (n^2 + 1)y^n$